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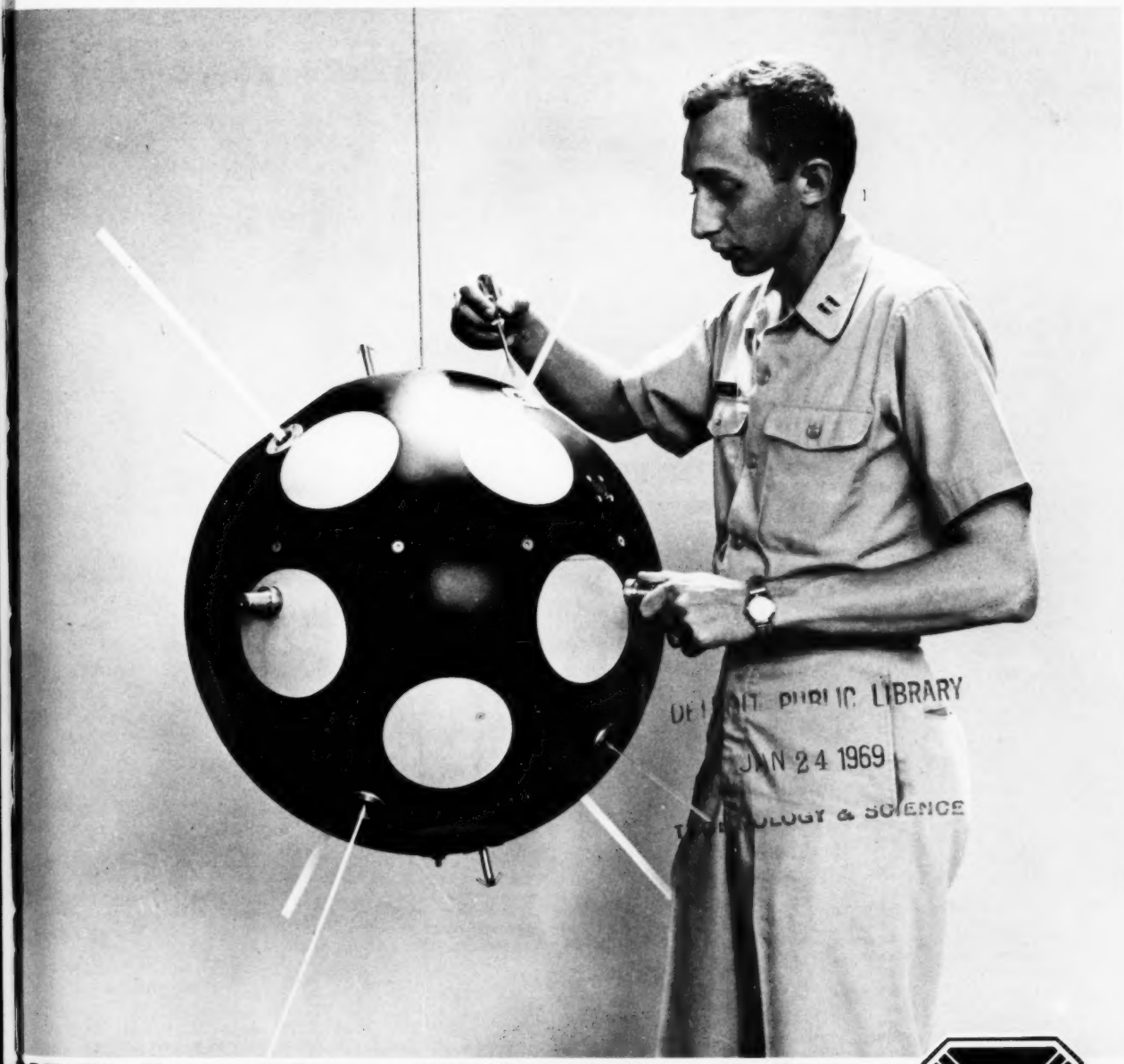
OAR RESEARCH REVIEW

OFFICE OF AEROSPACE RESEARCH ★ UNITED STATES AIR FORCE



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OAR RESEARCH REVIEW

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ON THE COVER:

The OV1-16 (LOADS/Low-Altitude Density Satellite), popularly known as the "Cannonball" satellite because of its spherical shape, was successfully launched by AFCRL on 11 July 1968 and re-entered the atmosphere on 19 August 1968. Its objective was to provide precise information on low-altitude atmospheric densities. Captain James D. Brown of AFCRL's Aerospace Instrumentation Laboratory is shown tightening an antenna mount on the satellite. (See article entitled, "Cannonball Satellite.")

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JACOB SEIDEN *Editor*

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report on the Conference on the Theory of Three-Particle Scattering in Quantum Mechanics*

DR. JOHN NUTTALL, Texas A & M University

Up to now, the only problem in quantum mechanics that we have been able to solve completely with any degree of precision has been the 2-body problem. This has brought us a long way towards understanding many different phenomena. However, there are many situations in which it is important to consider the interactions of 3 or more particles simultaneously and, because it is simplest, most attention has been turned to the 3-body problem. There has been a surge of interest in this problem in the present decade; and it was to discuss the recent work and to talk about hopes for the future that about 50 experts from the U.S.A. and Western Europe gathered at the Conference on the Theory of Three-Particle Scattering in Quantum Mechanics, held at Texas A & M University last April.

There are three main areas of physics where the three-body problem arises and, in each, the problem posed to the theorist is different. In atomic physics we know the equations that describe the system; the only question is how to solve them. In nuclear physics, we believe that the nonrelativistic Schrödinger equation provides a reasonable basis for the problem, but we do not know precisely what interactions to use. This uncertainty, added to errors in the solution of the equations, must be taken into account when calculations are compared with experiment. The dynamics of elementary particles should be treated relativistically, which adds to the difficulties; but we are not sure just which equations to solve. In any case, we expect that the motion of three particles cannot be treated as an isolated problem, but must be coupled to the motion of other numbers of particles. Thus we can be reasonably sure that any numerical calculations on systems of three elementary particles can only be approximate.

An important part of the Conference was devoted to a discussion of the various techniques employed to obtain numerical solutions to the equations describing the motion of three particles. Before standard methods can be applied, we must first overcome some mathematical difficulties not present in the two-body problem. These were emphasized by L. D. Faddeev, who approached the problem from the point of view of integral equations. Here, even if we are interested in three-body bound states, we must deal with a kernel which describes processes in which three free particles go to another state of three free particles. In this situation, a scattering event can occur in which one of the particles does not take part, and the disconnected nature of this process gives rise to unpleasant kernels not suitable for numerical approximation. Faddeev showed how to overcome this problem, and obtained integral equations with well-behaved kernels which could, in principle, be solved numerically.

In practice, simplifications must be made before the integral equations can be solved. These have been based on the idea of using separable potentials, suggested originally by R. D. Amado and A. N. Mitra, which reduces the problem to one-dimensional form. Dr. Amado (Pennsylvania) told the conference how excellent agreement with experimental n - d scattering results up to 14 MeV is obtained from calculations using a very simple separable potential chosen to fit the low-energy nucleon-nucleon data in $J = 0^+, 1^+$, states only. However, before we can make precise calculations of the properties of the three-nucleon system, we need to know the two-body interactions and, particularly, their off-energy-shell behavior, more accurately. This was pointed out in a detailed analysis by Dr. H. P. Noyes (SLAC) who remarked that, at present, we do not know the two-body interaction well enough to detect the existence of possible three-body forces.

The Conference also heard from Dr. D. Y. Wong (UCSD) about his work on extending the Amado-Mitra technique by working out separable approximations to the two-body t-matrices that appear in the Faddeev equations. Again the equations reduce to one-dimensional form. Wong described the result of calculations on systems such as (e,e,p) and three alpha particles. Also noteworthy was the contributed paper of Dr. P. E. Shanley (University of California, Davis) who applied separable techniques to the (p,n,α) system with excellent results.

An alternative approach to the solution of the three-particle problem is to work from the partial differential equation to which must be added suitable boundary conditions. These are well known for bound states and in the region below the breakup threshold, where the only open channels consist of two bodies (one a composite). Variational methods have proved to be a very effective way of solving the differential equation. Dr. L. Spruch (N.Y.U.) gave a comprehensive review of the present status of this field. In atomic problems, binding energies can be calculated to from 10 to 12 significant figures, and electron-hydrogen atom scattering, when just a few channels are open, is quite well understood. Theory was ahead of experiment in predicting a series of resonances below each threshold.

In a variational calculation a major part of the effort goes into an evaluation of the matrix elements of the Hamiltonian between a set of expansion functions, and the integrals involved can be done analytically in the case of Coulomb potentials. From this point of view the atomic problem is significantly easier than the nuclear case, where "realistic" potentials are

The Conference on the Theory of Three-Particle Scattering in Quantum Mechanics was held at Texas A & M University, 4-6 April 1968. Dr. Nuttall is an AFOSR grantee, and is supported by the Nuclear Physics Division, AFOSR.

so complicated that the integrals must be done numerically. The participants heard Dr. L. M. Delves (Sussex, England) describe the labor that ensues when one tries to determine the triton binding energy for a nucleon-nucleon potential, such as the Hamada-Johnston. As improvements are made to the trial wave function, the estimate of the binding energy increases; but it appears likely that, even after several years' more work, it will still be significantly less than the experimental values. Delves' talk was notable for giving the Conference an insight into some important technical details, very helpful to anyone contemplating a similar calculation.

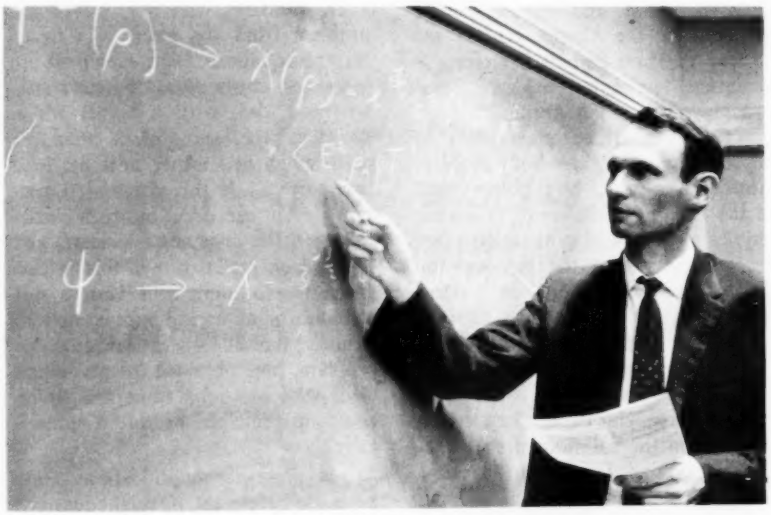
In the relativistic domain Lagrangian field theory, leading to an expansion in terms of Feynman diagrams, may yet contain a description of nature; but our inability to sum the expansion stops us from finding out. The hierarchy of generalized Bethe-Salpeter equations provides a way of summing the diagrams and, recently, interest has grown in the solution of such equations with a restricted version of the irreducible kernel. The result gives the sum of a limited class of Feynman diagrams. Dr. C. Schwartz (Berkeley) told the Conference about his ingenious numerical solution of a three-body problem of this type where, for simplicity, the total center of mass energy was zero. He thought it would be possible to extend his methods to higher energies.

Schwartz has not yet applied his ideas to physical problems; but Dr. R. L. Omnès (Orsay, France) reviewed some less sophisticated calculations of Basdevant and Kreps on the three-pion system. The propagators in the nonrelativistic Faddeev equation are modified to make them covariant, and a separable two-body interaction is used, which amounts to assuming that pion-pion scattering is dominated by the rho-resonance. The calculation predicts a number of resonances, some of which may correspond to observed effects, but does not explain the omega-resonance.

There is an alternative approach to the dynamics of elementary particles based on on-mass-shell S-matrix theory, which makes extensive use of the analyticity properties of the scattering amplitude. One way of determining what these properties are is to generalize the nonrelativistic results. With this in mind, Dr. R. L. Sugar (UCSB) told the Conference about the work he and his collaborators have done to study the analyticity of three-particle scattering amplitudes when the interaction is formed from a superposition of Yukawa potentials. He mentioned some new results about dispersion relations in the momentum transfer variable for particle-bound state scattering. Also, in the area of S-matrix theory, Omnès reviewed the N/D approach to the three-body problem, and considered in detail the computation of form factors. He reported that the Frazer-Fulco analysis can be extended to three-particle intermediate states, but that difficulties arise when more particles are included.

Although significant progress has been made in the three-body problem, many important questions remain. These were touched upon by Dr. R. Blankenbecler (UCSB) in a summary of the Conference. There has not been much calculation for energies above the breakup threshold, where additional singularities must be dealt with in the Faddeev equation. From the differential-equation point of view, the boundary conditions are more complicated, although the author has recently shown that they can be obtained in usable form for the case of short-range forces. Even the threshold behavior at breakup is not fully understood. In the case of Coulomb interactions, the difficulties in the region above this threshold are compounded by the long-range forces. This classic problem, important to the study of ionized gases in space, remains unsolved.

This Conference appears to have been the first devoted solely to the three-body problem, but is not likely to be the last. I believe the participants will join me in thanking the Nuclear Physics Division of the Air Force Office of Scientific Research and Texas A & M University whose support made the Conference possible.



Dr. John Nuttall, born in England, was educated at St. John's College, Cambridge, where he received First Class Honors in Part II of the Mathematics Tripos, and a Distinction and share of the Mayhew prize in Part III of the Mathematics Tripos. After receiving his B.A. in 1957, he continued as a research student in theoretical physics at Cambridge, working on nucleon-nucleus scattering and dispersion theory under Dr. R. J. Eden. He was awarded the Smith's prize in 1959 and spent the year 1959-60 at Princeton University as Procter (sic) Visiting Fellow, receiving the Cambridge Ph.D. degree in 1961.

Dr. Nuttall returned to Cambridge for two years as NATO Research Fellow and Fellow of St. John's College. In 1962, he moved to the RCA Victor Company in Montreal, Canada, where he worked on the theory of plasmas, with particular reference to the ionosphere and space. He came to Texas A & M University in 1964 as NSF Senior Foreign Scientist, and is now Associate Professor of Physics and principal investigator for an AFOSR theoretical-physics research grant. His present research interests include several aspects of scattering theory and the many-body problem.

cannonball satellite

AERONOMY LABORATORY, AFCRL

A small spherical satellite (see cover photo) having a diameter of only 23 inches and weighing 600 pounds was launched by AFCRL on 11 July 1968, from Vandenberg AFB, California aboard an Atlas booster. Its apogee was 310 NM and its perigee was 84 NM. It re-entered the earth's atmosphere on 19 August 1968, near Adelaide, Australia. There is no evidence that any portion of the satellite reached the earth.

This satellite, the OV1-16 (or LOADS/Low-Altitude Density Satellite), was popularly known as the "Cannonball" satellite because of its spherical shape. It had a density of 162 pounds per cubic foot, a density much higher than that of any other satellite previously launched. The operation of the OV1-16 was very successful, providing over 200 data acquisitions.

The cannonball satellite was designed to provide the most precise and detailed information to date on atmospheric densities in the altitude region between about 80 and 200 miles. The high mass-to-area ratio of the satellite was intended to reduce the deceleration due to atmospheric drag, permitting it to remain in orbit for a useful period despite its very low perigee.

The primary instrumentation aboard the satellite was a specially designed triaxial acceleration-sensing system. The values of subtle decelerations provided by the sensors, when combined with the satellite's mass-to-area ratio, drag coefficient and velocity, were intended to provide a continuous measure of atmospheric density. From measurements of density, it is possible to derive pressures and temperatures at the satellite altitude.

Knowledge of atmospheric densities in the 80-200-mile region will permit greater accuracy in predicting satellite orbits and splash-down points of re-entering space vehicles. Of particular interest are short-term fluctuations in atmospheric density. Weather patterns, analogous to those experienced at ground level, exist at satellite altitudes. Pressure, density, and temperature vary with altitude, latitude, longitude, and time. The continuous short-time resolution measurements made by the satellite will provide data for weather charts of the atmosphere above 80 miles.

The success of the mission was in great part due to the acquisitions at Ft. Churchill. This station not only provided data during the period when the perigee was in its vicinity, but also, because of its location, allowed collection of data from successive stations.

The satellite was designed by Dr. Kenneth S. W. Champion of AFCRL, who is responsible for the over-all AFCRL atmospheric-density-measurement program.

Dr. Champion using an oscilloscope to check out instrumentation for the "cannonball" satellite.

Dr. Kenneth S. W. Champion has been Chief of the Atmospheric Structure Branch, Aeronomy Laboratory, AFCRL since 1963, and a full-time AFCRL staff member since 1959. He supervises about 30 civilian employees with a wide range of programs concerned with the ionized and neutral properties of the upper atmosphere and nearby space.

Dr. Champion received his B.S. with Honors in Physics and Mathematics from the University of Sydney, Australia, in 1944, and his Ph.D. in Electron Physics from the University of Birmingham, England, in 1951. In 1947 he was awarded a British Council Travelling Scholarship and, from 1949-1952, he held a research fellowship from the Australian National University. He was appointed Assistant Professor of Physics at Tufts University in 1954. During part of 1964 he was Visiting Professor of Physics at the University of Adelaide, Australia.

Dr. Champion has received several awards. He is a member of the American Physical Society, the Physical Society of London, RESA, and the American Geophysical Union.

THE LOCOMOTION QUADRUPED

LT COL ROBERT B. KALISCH
Directorate of Engineering Sciences, AFOSR

Automata or automatic machines have long been in the realm of science fiction. In fact, the association between robots and science-fiction literature is so strong that one intuitively doubts the importance of research related to automata. Notwithstanding the emotional response provoked by such work, Professor R. B. McGhee of the University of Southern California (USC) has studied a locomotion quadruped for several years. (See Figure 1.) This work has been supported by AFOSR through the Joint Services Electronics Program (JSEP).

Unlike many such proposed designs, this locomotion quadruped employs finite-state logic. An elementary finite-state machine can be made by permitting each leg to have only two output states: on the ground pushing backward, and in the air moving forward. If each leg is controlled in this way, the over-all sequencing of the motors powering each joint can be accomplished by a small digital computer. In reality the computations are so simple that the computer is nothing more than a binary sequence generator. It determines the next motor to be activated and the polarization of the voltage to be applied to it.

The use of finite-state logic has allowed a more satisfactory treatment of stability questions for certain types of gaits. In a human being, for example, with each limb operating more or less independently, an infinite number of states would have to be studied to treat the stability question analytically. This situation arises because one must describe not only the current position of each limb but also the history of past positions. By contrast, a sequential finite-state machine always achieves the current state in a specified manner. Therefore, stability considerations are governed by the entry into the state and conditions in the state—a greatly simplified problem.

Professor McGhee designed the USC automaton to have a locomotion capability in several gaits. Each gait, e.g., crawl, walk, trot, etc., requires a different program for the sequence generator. Dependent upon the degree of sophistication desired, the number of states for each movable part is determined. A synthesis program such as this provides a logical basis for a design of locomotion automata. Unfortunately, the design procedure is not developed to the point where one can insure stability for every type of gait.

Several dissimilar motivations encourage this research. Perhaps the dominant one is the desire to design a system whose locomotion capabilities would exceed those of conventional wheeled vehicles. The improved locomotion capability would undoubtedly be limited to special environmental conditions such as marshy terrain, deep snow, or very irregular surfaces, including mountain terrain. Certainly, the major impetus for locomotion automata stems from such motivations. Another motivation is the wish that, from design experience with quadruped automata, one could generalize techniques applicable to prosthetic biped devices. Such an application, while far from being realized, does not seem impossible. Significant progress has been made by Professor G. A. Bekey, whose work at USC has also been supported by AFOSR through JSEP. In his research, paraplegic animals have been able to move their limbs in response to stimuli from a binary sequence generator grossly similar to that used in the locomotion automaton. Many questions still remain before the complete transfer of the technique from machine to animals reaches fruition.

Hopefully, the binary character of the sequence generator of the automaton could be copied in animals to distribute appropriate pulses to specific muscles. If the muscles could be caused to contract and extend by pulse stimulation, normal limb motion could be simulated. If this intermediate goal is completely realized, one could then address oneself to the exceedingly difficult problem of the stability of animals while in motion. The complexity of this problem is in part related to the variety of muscles which, when flexed by an animal, would alter the center of gravity or produce an unpredicted moment which might cause the subject to fall. A pragmatic advantage resulting from the use of a biological subject is the adaptation or learning process. While this could serve to assist an animal in the use of a prosthetic device, from an analytical viewpoint it is a most intractable characteristic.

The military has still another motivation for this research. In many autopilot or remote-controller applications an understanding of how a human responds to a stimulus is essential to the design of a system. Usually, research in this area is directed at the instrumentation of a human and the recording of a variety of skin potentials, blood pressures, eye movements, limb movements, etc. The research at USC could be considered an attempt to derive the minimum elemental impulse within an animal to cause a particular muscular reaction.

As these dual series of experiments have evolved over the past several years, there has been an increasing industrial interest both in a locomotion quadruped and in animal analogs. Proposals have been made to DoD to begin work on the design of a locomotion quadruped for use in remote areas. Specifically mentioned in this regard are the problems posed by crevices in the ice caps of the polar regions, lunar terrain (probably no longer a problem), and the rice paddies of Southeast Asia. The nature of the ultimate application of this research is not of dominant concern to the basic-research community. However, this basic research—phenomena-oriented and application-motivated—does have a high probability of significance for a number of DoD activities.

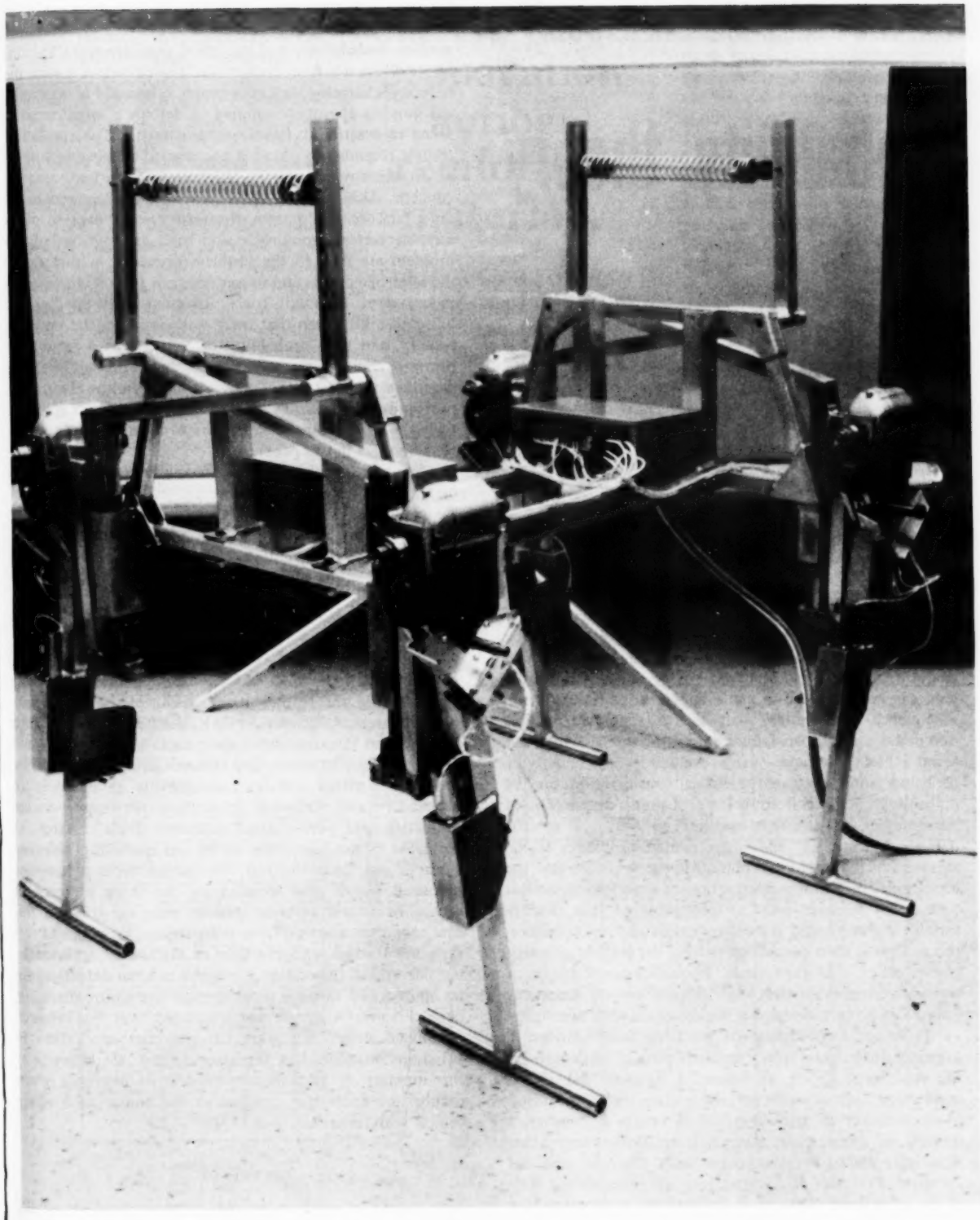


Figure 1. Locomotion quadruped under study at the University of Southern California.

MAJOR WILLIAM R. TROTT
Directorate of Mathematical Sciences, AFOSR

probability theory in weapon-system evaluations

Probability theory and its many subareas play an important role in numerous considerations involving weapon systems and weapon-system effectiveness. Many of the problems that arise in attempting to evaluate a weapon system or to compare its effectiveness with other weapon systems can be formulated into probabilistic expressions for analytic solutions and analysis. Furthermore, many problems can be structured so that events to be considered form a sequence of Bernoulli trials—that is, only two possible outcomes, success or failure, are considered. While this setup is basic in weapon-system evaluations, the underlying probability and associated statistical considerations may be difficult. Of major concern is the probability distribution of the number of attempts required to obtain a specific number of successes and other related problems. If the structure of the stochastic (random) process is such that the conditional probability distribution of a random variable at time $n+1$ depends only on the value of that random variable at time n and is independent of all previous values, the process is then considered to have the Markov property and is called a Markov chain. In particular, problems of weapon-system evaluation with concern for only success or failure can be considered as a Markov chain with two states.

Professor J. S. Rustagi of the Ohio State University, principal investigator on a research project supported by the Air Force Office of Scientific Research (OAR), is conducting basic research into the waiting-time distribution of the number of trials required to obtain a preassigned number of successes in a sequence of Markov-dependent Bernoulli trials. In connection with this, he and his associate, Professor R. C. Srivastava, are investigating tests of an hypothesis concerning appropriate parameters that might enable one to develop decision procedures for choosing more effective weapon systems.

In a recent paper entitled "On a Distribution Arising in Weapon Systems,"(1) Professors Rustagi and Srivastava present probabilistic and statistical considerations applicable to weapon-system evaluations. Their concern is with the lethality characteristics of weapon systems. For example, the number of rounds required to destroy a target may be taken to describe its lethality characteristic. The number of rounds required to obtain a preassigned number of hits so as to destroy the target is another important area of concern. Although the above paper, as written, was based on a tank-weapon system, the results carry over to other weapon systems so long as the basic assumptions of the problem are met. In the problem discussed in this paper, the sequence of hits and misses does not form a sequence of independent Bernoulli trials. Assuming that the "probability of a hit, given that there was a miss on the previous round," and the "probability of a hit, given a hit on the previous round" remain constant, these probabilities are described with the aid of a two-by-two Markov chain. The probability distribution of the number of rounds required to get a specified number of hits to insure target destruction is shown in this paper.

Statistical problems associated with the above probability distribution are also investigated. An estimation of the parameters involved is discussed, and an application of this model for the tank-weapon system is considered. Some values of the distribution are tabulated. Although the distribution arises naturally in applications of the above type, the results of the paper have much wider application such as in transportation engineering, computer failures and queueing theory.

Probability and Statistics are two major areas of mathematics in which basic-research grants are being supported by the Mathematics Division of the Directorate of Mathematical Sciences, AFOSR. Many areas of Air Force concern and relevance have their roots in probability and statistical considerations. The research being conducted by Professor Rustagi and his colleagues is an example of probability and statistical application to weapon-system evaluation and other related problems. Their research, as indicated by the description above, has application to many areas of Air Force interest. Similar problems to the one discussed above arise in studying the firing patterns of ground-to-air and air-to-air missiles since the trials in this case cannot be assumed to be independent. Problems of this type are studied with the help of Markovian dependence considerations. In addition, problems in sonar detection can be approached through techniques of the nature discussed here. Finally, it should be mentioned that the research conducted under this grant has potential application to situations introduced by the space age, e.g., the behavior of the number of adjustments needed to obtain a given number of successful changes in the course of a space vehicle with finite fuel.

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elastic wave propagation in fiber-reinforced materials

1ST LT MARION G. POTTINGER

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An examination of projected space research efforts indicates that space probes, upon re-entering the earth's atmosphere, will encounter gas-dynamic phenomena not experienced by earthbound air vehicles or even re-entering earth satellites. The gas-dynamic effects encountered at velocities of 30,000 to 150,000 ft/sec cannot presently be duplicated.

One Fluid Dynamics Facilities Research Laboratory contract effort (Ref. 1) has the goal of developing an implosion-driven hypervelocity launcher capable of accelerating sizeable aerodynamic models to velocities of more than 35,000 ft/sec. A major problem in achieving the aims of this program is the production of models which will retain their shape during launch.

During a launch, a model would typically be subjected to a forcing function such as that shown in Figure 1, which is taken from Sevray (Ref. 2). From data such as that in Figure 1, curves showing average base pressures during launch as a function of projectile specific gravity can be drawn (Fig. 2). If typical material ultimate strengths are plotted as points on the same graph as the average base pressures, it is possible to conclude that those materials combining high strength and low density will allow launches to the highest velocities. For example, in Figure 2 it can be seen that 50% B in Mg and high-strength steels have comparable strengths. Also from Figure 2 it can be seen that the velocity associated with the base pressure equivalent to the material ultimate strength is about 30,000 ft/sec in the case of 50% B in Mg, and about 17,000 ft/sec in the case of high-strength steel. This type of comparison indicates that higher launch velocities will be a direct function of projectile-material efficiency. Therefore, the most structurally efficient materials, fiber-reinforced composites, will be the model materials necessary to realize the greatest launcher performance.

The dynamic properties of fiber-reinforced materials are not well enough known to permit a rational analytical approach to model design (Refs. 3, 4, 5, and 6). Therefore, an in-house effort, adequate to define their dynamic properties to an extent permitting rational model design, has been undertaken. The effort is a three-step approach to the dynamics of fibrous composites in the elastic range.

The first step is an attempt to ascertain the validity of anisotropic elasticity theory as a mathematical model of the dynamic elastic behavior of fiber-reinforced materials. The approach combines both experiment and analysis, with emphasis given to the dispersion characteristics of the material. The elastic constants utilized in the theoretical portion will be based on statically derived constants, such as those given in References 7 and 8. Both metal-matrix and polymer-matrix

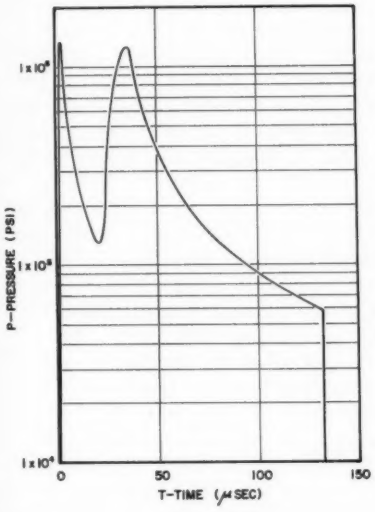

Figure 1. Pressure-time history for the launch of a 1-caliber, 1-inch-diameter projectile of specific gravity 1, to 15 km/sec for the 30-inch chamber launcher.

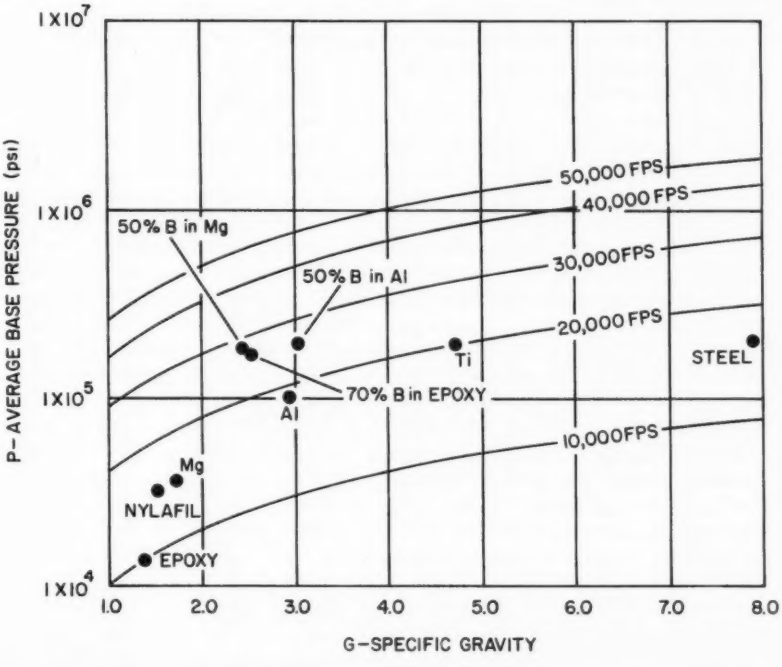

Figure 2. Average base pressure and ultimate material strengths as a function of launch velocity and projectile specific gravity.

materials will be studied.

The second step is the production of computer codes capable of describing wave propagation in two-dimensional, anisotropic, elastic solids.

The third step will be the application of the work of step two to the model-design problem if step one shows that the anisotropic elasticity theory is a valid model of the dynamic behavior of composites. If step one does not confirm the model, then step three will become a search for a better model.

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1st Lt Marion G. Pottinger is a research engineer with the Fluid Dynamics Facilities Research Laboratory, ARL. His basic interests are in applied stress analysis, the dynamics of machines, structural dynamics, and dynamic simulation using computers. In 1961 he received a B.S.M.E. from the University of Cincinnati. Subsequently, he received an M.S.M.E. in Machine Design (1963) and a Ph.D. in Dynamics and Vibration (1966), both from Purdue University. While at Purdue he not only conducted research, but also taught part of, and organized the laboratory equipment for, "Measurements and Computers," a junior-level course in the Mechanical Engineering Department. Lt Pottinger is a member of the American Society of Mechanical Engineers, the Society for Experimental Stress Analysis, and Sigma Xi, and has several publications to his credit.

COMMUNICATIONS IN QUANTUM CHEMISTRY

LT COL EDWARD T. WALFORD, Directorate of Chemical Sciences, AFOSR

The field of quantum chemistry is closely related to quantum physics or quantum mechanics. By using quantum theory to ascribe wave-like properties to electrons, chemists have gained new insights into the nature of matter. Some believe, hopefully, that quantum theory will eventually provide the unifying principle for the understanding of the whole of chemistry. This is the ultimate objective for research in this vital area of chemistry.

AFOSR has a carefully selected group of investigators in this difficult, complex, way-out field of research. Included are Dr. Harrison Shull of Indiana University, Dr. Michael Dewar of Texas, Dr. Leland Allen of Princeton, and Dr. Andrew Streitwieser, Jr. of California. However, it is not our purpose in this article to further describe their important work but to discuss the communications problem in quantum chemistry and one effort at its solution.

In a recent article(1) Dr. Per-Olov Löwdin discussed the often-heard fallacy that quantum mechanics, with the help of sufficiently large electronic computers, should be able to completely replace experiments in physics and chemistry. In reality quantum mechanics is essentially the tool for handling the experimentally determined information about a given system. The theory should then produce the fundamental principles for understanding the system, once the system has been described by the proper experimental information. This is shown schematically by Figure 1, taken from Dr. Löwdin's article.

Unfortunately, quantum chemistry is more of a conceptual framework than a nice set of mathematical laws. In advancing this area, a horrendous effort must be spent on numerical analysis and numerical computation. The importance of electronic computers to the development of quantum chemistry cannot be overemphasized. Problems related to the electronic structure of atoms, molecules, and crystals lead to computations of a formidable order of magnitude. It is only after thorough numerical analysis and reduction that the present problems can even be treated by today's computers.

Dr. Robert S. Mulliken, formerly an AFOSR investigator, addressed this problem in his recent acceptance speech (2) for the 1966 Nobel prize in chemistry:

> "I would like to emphasize strongly my belief that the era of computing chemists, when hundreds if not thousands of chemists will go to the computing machine instead of the laboratory for increasingly many facets of chemical information, is already at hand. There is only one obstacle, namely, that someone must pay for the computing time. However, it seems clear that the provision of adequate funds by the Government and other organizations for computing molecular structures has at least as high an order of justification as the provision of adequate funds for the cyclotrons, bevatrons, and linear accelerators used in studying

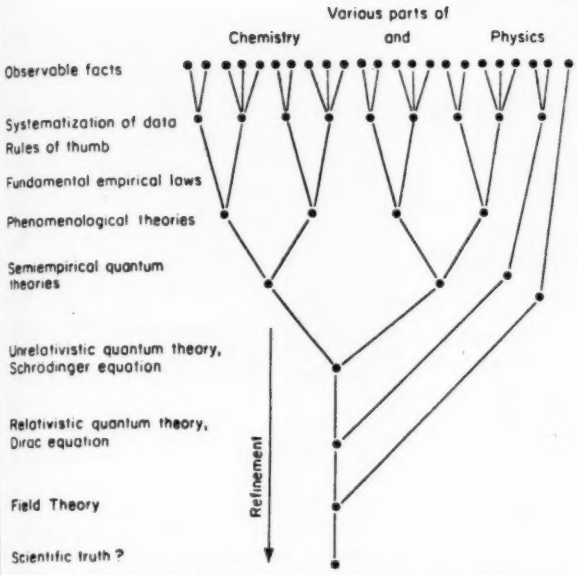

Figure 1. Schematic diagram of the hierarchy of theories concerning the structure of matter, showing the increase of correlated areas of observable facts with the refinement and deepening of the basic principles.

> nuclear structure and high-energy particles, or for rockets to explore the moon, planets, and interplanetary space. Chemistry, together with the physics of solid matter on the earth, deals with the foundations of the material world on which all our life is built."

The Directorate of Chemical Sciences, AFOSR, is presently engaged in a key effort to alleviate the communications problem for chemists by its sponsorship of the Quantum Chemistry Program Exchange (QCPE). This program, and its historical development under Dr. Harrison Shull at Indiana University, may be explained as follows:

The efforts of scientists to understand the chemical properties of atoms and molecules from first principles, that is, as based on quantum mechanical calculations, is as old as quantum mechanics itself. Both efforts date from the late 1920's. The high point of understanding in the precomputer era was reached somewhere in the early 1930's with the work on the hydrogen-ion molecule and the hydrogen molecules. This work represented the upper limit of scientists with regard to making calculations. This situation prevailed up through the mid-1950's.

In the later 1950's, the first computers of any significance became available and, with the advent of these computers, possibilities for making meaningful calculations were also greatly increased. However, concomitant with this increase in possibilities was a requisite increase in the investment of effort and manpower in the task of communicating with these computers. The traditional approaches which had dominated the pre-1958 period were of little value in the computer era. Problems had to be specifically formulated for the computer. As part of this formulation, new techniques of numerical analysis had to be simultaneously developed. It was now necessary to code the problem to some form which the computer understood, i.e., the computer program; and finally, the program had to be made error free—debugged—and certified as operational. All of this required time and money.

As the desire to understand larger systems from *ab initio* calculations became prevalent, it was not an uncommon practice for research groups to spend a year or more in the preparation of computer programs. It was immediately recognized by many workers in the field that, in many cases, duplicate efforts were taking place. It was also obvious that many groups were developing calculations that would be of distinct interest to many other groups. The early workers in this field wanted to utilize their resources, i.e., manpower, money and computer time, in an optimal fashion.

With this goal in mind, Dr. Shull agreed to establish, at Indiana University, what is known today as the Quantum Chemistry Program Exchange (QCPE). Dr. Shull recognized that what was needed was a means of communication between workers in the field as well as a central point to which they could turn in order to obtain dependable computer programs.

In operation then, the QCPE relieves the investigators of many tedious hours of laborious work by providing tested computer programs upon request for a small handling fee. It serves as a central repository for these programs which are solicited from the contributing members. The members receive a quarterly newsletter (3) which contains abstracts of the available programs. At the beginning of this program in 1964, there were 135 participants and 30 programs in the exchange. Today the exchange lists about 800 participants and 110 programs.

The QCPE requires each participant to reply to an annual questionnaire. This guarantees a minimum of genuine interest in the exchange and provides valuable feedback information. For example, the participants have shown a continuing increase in computer time on large systems. The savings on this computer time, even if only 1%, are more than sufficient to pay for the program. But, more importantly, the general availability of these programs saves enormous time in duplicative efforts. It permits the individual investigators to concentrate more directly upon quantum chemistry itself and, therefore, to produce more significant research.

A recent voluntary check of the participants showed that the QCPE was cited as the source of computer programs in 49 journal articles, mostly in the 1965-1966 time period. Data of this type show that the QCPE is making an impact and is helping to relieve the shortage of computer funds. It is also evident that, in sponsoring the QCPE, AFOSR is helping to significantly advance an important research area of chemistry.

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self-diffusion of radioactive tracers across discontinuous surfaces

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In recent papers (Refs. 1, 2, and 3), the authors have analyzed the effect of a discontinuous diffusion couple interface on the experimental determination of the self-diffusion coefficient in solids. A diffusion couple is produced by pressing a radioactive pellet against a nonradioactive pellet where contact over the entire interface is difficult to achieve. The analytical studies examined the three-dimensional diffusion across a single contact spot to determine if one-dimensional theory could be used to determine the diffusion coefficient from experiments if the actual contact area were used. Linder (Ref. 4) had employed this technique, but provided no theoretical basis for its validity. He determined the contact area by comparing the measured electrical resistance of the diffusion couple with the resistance of a continuous pellet of equal size. In the experiments, the diffusion couple is subjected to a high-temperature anneal, and the redistribution of the radioactivity is measured. The diffusion coefficient is then determined from the relation

$$D = \frac{\pi\sigma^2}{A^2 C_o{}^2 t} \qquad (1)$$

where σ is the total radioactivity that has crossed the pellet interface during the anneal time t, C_o is the initial concentration, D is the diffusion coefficient, and A is the contact area. This relation is obtained from the one-dimensional solution of the diffusion equation

$$\frac{\partial C}{\partial t} = D\nabla^2 C \ . \qquad (2)$$

In the author's studies, equation (2) was solved for the three-dimensional concentration distribution in the upper half of the idealized contact module shown in Figure 1. The interfacial gradient across the contact spot was then determined to be

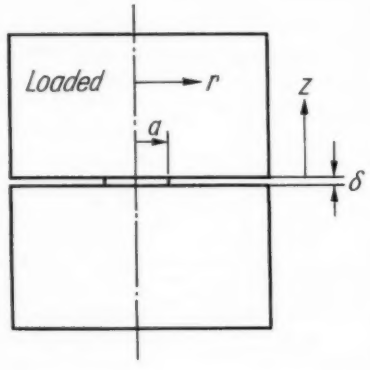

Figure 1. Cylindrical contact module.

$$\left.\frac{\partial C}{\partial x}\right|_{x=0} = \frac{C_o a}{2\sqrt{\pi Dt}} \int_0^\infty J_1(ap)\, J_o(rp)\, dp \qquad (3)$$

$$= \frac{C_o}{2\sqrt{\pi Dt}} \quad 0 \le r \le a$$

$$= 0 \quad r > a$$

where a is the radius of the contact spot and the J_i are Bessel functions of the first kind. The total material transport in the time t is

$$\sigma = 2\pi D \int_o^t \int_o^a \left.\frac{\partial C}{\partial x}\right|_{x=0} r\,dr\,dt \qquad (4)$$

$$= C_o \sqrt{\pi Dt} \quad a^2$$

and then $$D = \frac{\pi\sigma^2}{A^2 C_o{}^2 t} \qquad (5)$$

This relation is identical to the one-dimensional result when A is the actual contact area and Linder's technique is verified.

Since diffusion across a contact surface is also an important problem in heat conduction, the concentration distributions were computed for several values of the parameter $\frac{a^2}{4Dt}$ and are shown in Figure 2 where $R = r/a$, $\xi = x^2/4Dt$ and $\theta = C_o - C$ is the concentration defect due to the depletion by the diffusion across the contact spot.

The results demonstrate that the influence of the contact spot is restricted to less than 10 spot radii in the radial direction, and less than 4 in the axial direction. Also, since a surface can be constructed with many contact modules, any experimental technique that averages the concentration in the plane perpendicular to the diffusion direction may be used to evaluate the diffusion coefficient with one-dimensional theory and the correct contact area. For example, the constant surface concentration and vanishing thin-film methods can be used even though the surface has a nonuniform accessibility or the thin layer is not continuous.

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Mr. Kenneth R. Cramer has been a member of the Aerospace Research Laboratories since July 1951. He is presently engaged in analytical investigations of plasma microinstabilities and plasma-containment problems. His past experience has been in theoretical investigations of various phenomena in flows of electrically conducting and nonconducting fluids, heat

transfer and tracer-particle diffusion, theoretical boundary-layer studies, radiative heat transfer, forced- and free-convection heat transfer, conductive heat transfer, magnetohydrodynamics, and thermal problems of orbit-type space suits. Mr. Cramer's experimental research experience has involved investigations of a traction pump and a supersonic compressor. He received his B.M.E. from the University of Dayton in 1951, an l his M.S. from Ohio State University in 1956. He is the author and coauthor of a great number of publications.

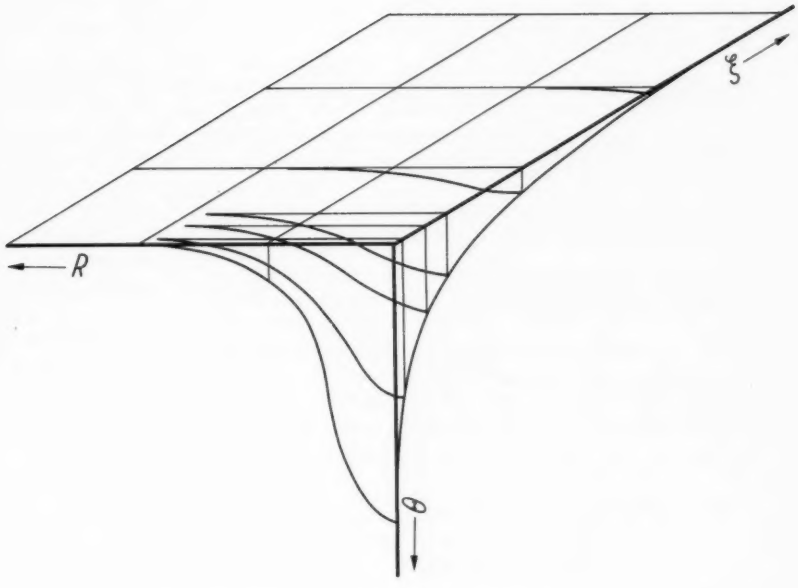

Figure 2. Contact-spot concentration distribution.

radiation damage in metals

SOLID STATE SCIENCES LABORATORY, AFCRL

The direct displacement of atoms from their normal positions in a crystal is one of the chief mechanisms responsible for property changes in irradiated materials. The electron bombardment of solids is a useful technique for the experimental study of displacement production. Firstly, the energy transfers are so low that only isolated Frenkel pairs (vacancies and interstitial atoms) are created--therefore, the observed property changes are directly related to the properties of these fundamental defects. Secondly, the energy can be lowered to the point where damage just starts to occur, thus allowing study of the threshold region for the displacement process. Studies of damage-production rates as a function of energy afford an experimental approach to many fundamental questions, for example:

How does the threshold energy for atomic displacement relate to other material parameters?

What are the type and magnitude of the property changes associated with point defects?

How many vacant lattice sites and interstitial atoms are created by a given radiation environment?

How many atoms are displaced by an energetic primary knock-on in an atomic cascade?

Some recent experiments carried out in this laboratory by E. A. Burke, C. M. Jimenez and L. F. Lowe on the face-centered cubic elements, platinum and palladium, furnish information that bears directly upon these questions. The results support the correlation of effective threshold energy with atomic number and lattice parameter for the face-centered elements. For these elements it was also found that the electrical resistivity per unit defect concentration can be correlated with the resistivity at 0°C. Finally, evidence for employing an atomic cascade model in which secondary displacements are inhibited will. be presented.

In experiments of the type to be described, the observed change in the electrical resistivity of the bombarded specimen is related to the incident electron beam by the expression

$$\Delta\rho/n = \sigma_d \rho_f \ ,$$

where

$\Delta\rho$ = the observed change in resistivity

n = the integrated electron flux

σ_d = the cross section for electron-atom interaction

ρ_f = the resistivity per unit defect concentration (UCFD).

The cross section σ_d is obtained from

$$\sigma_d(E) = \int_0^{T_m} P(T)\ \nu(T)\ \left(\frac{d\sigma}{dT}\right)\ dT\ ,$$

where

T_m = the maximum energy that can be transferred to the struck atom

$P(T)$ = the probability that an atom which receives energy T will be displaced.

For the purposes of this discussion, we shall assume a step probability function, i.e., P(T) is taken as 1.0 for all values of T greater than T_d and zero for all values of T less than T_d. T_d will be referred to as the effective threshold for atomic displacement and replaces the lower limit 0 in this equation. We have studied the influence of P(T) on damage-production rates and found that, for any reasonable function, the effects are small for the energies employed here.

$\nu(T)$ = the number of displaced atoms per primary displaced atom. This will be discussed in detail later.

$\left(\frac{d\sigma}{dT}\right)$ = the differential cross section for electron-atom interaction. Because of the high Z of Pt and Pd and the relativistic electron energies used, we used the Mott-Rutherford formula for $\frac{d\sigma}{dT}$ rather than approximations, such as that of McKinley-Feshbach, for this calculation.

As stated before, we measure $(\Delta\rho/n)$, and therefore only the product $\sigma_d\rho_f$ is determined experimentally. However, if it is assumed that ρ_f does not change as a function of bombarding energy, experimental values of $\Delta\rho/n$ can be compared with various theoretical cross sections derived by using assumed values of T_d. Where multiple displacements are important, different models for $\nu(T)$ can be used and tested. At low energies $\nu(T)$ can be taken as unity.

The Pt used had a room-temperature to liquid-He-resistivity ratio of 230 and, for Pd, this ratio was 950, both for the mounted samples. The damage rates were determined at liquid He temperature in order to "freeze" the defects in and also to suppress the background signal.

In the case of Pt, when theoretical curves for different T_d are compared to the experimentally determined damage rates, the best fit yields a T_d of 37 ev and also a ρ_f of 7.5×10^{-4} Ω cm/UCFD. At this point it is interesting to note that, at the higher energies where multiple displacements should be just becoming important, our damage rate starts to tail off. Since it was felt that it would be interesting to see if this was a real effect, we decided to try an element of lower A which would allow us to go further into the multiple-displacement region.

We decided to use Pd for this purpose since its atomic weight is about 1/2 that of Pt. It was expected that 3.0 MeV electrons would produce multiple displacements in Pd. This was thought to be the case since a 3.0-MeV electron can impart ~ 230 ev to a Pd atom or, in other words, $T_m \cong 7\text{-}8\ T_d$, assuming a T_d of about 30 ev for Pd. To interpret damage rates in this region a model for $\nu(T)$ must be used. We utilized an approach to the formulation of $\nu(T)$ which was general enough so that the commonly used cascade models were special cases of this approach. We found that, by defining two quantities T_s and T_ℓ, we could characterize not only the existing models but also many variations of these models. The quantity T_s is the amount of energy required for a knock-on atom to have, after it displaces an additional atom, in order to escape from the lattice site of the new displaced atom. In other words, if it has less than T_s, a replacement will occur. The other quantity, T_ℓ, is the energy that the displaced atom loses to the lattice in the displacement process.

Utilizing various combinations of T_s and T_ℓ, we were able to test various multiple-defect models. We used a weighted least-squares fit to compare the experimental results with the models. The result was that the model, which predicted no multiple displacements, gave the best fit to the experimental data even though $T_m = 7\ T_d$ for 3.0-MeV electrons. The model which predicted the greatest number of displacements gave the poorest fit, with other models falling in between. This analysis led to a value of 34 ev for the T_d and a ρ_f of 9×10^{-4} Ω cm/UCFD.

One further point on the applicability of the various multiple displacement models to the face-centered cubic metals: recent results of electron irradiations on Al and Cu show good agreement with the Seitz-Koehler and Kinchin-Pease models; however, our results on Pd are better explained by a simple displacement model with no, or very low, secondary displacement production. The limited data on Pt in the multiple-displacement region also suggests agreement with such inhibited multiple-displacement models. This suggests that the production of multiple displacements in intermediate to high Z fcc metals may be inhibited in contrast to recent results on low Z elements (Al and Cu). More experimental work on high Z metals at energies which penetrate deep into the multiple-displacement region would be desirable to clarify this point.

SUMMARY

To summarize our results we have found the following:

1. For Pt an effective T_d of 37 ev was measured along with a corresponding ρ_f of 7.5×10^{-4} Ω cm/UCFD.
2. For Pd an effective T_d of 34 ev was found along with its ρ_f of 9×10^{-4} Ω cm/UCFD.
3. The data for Pd agrees with the single-displacement model even though the maximum transferred energy is about 7 times T_d.
4. The production of multiple displacements for intermediate to high Z fcc metals may be inhibited.

Two Modes of Momentum Transfer for a Stationary Arc in a Transverse Magnetic Field

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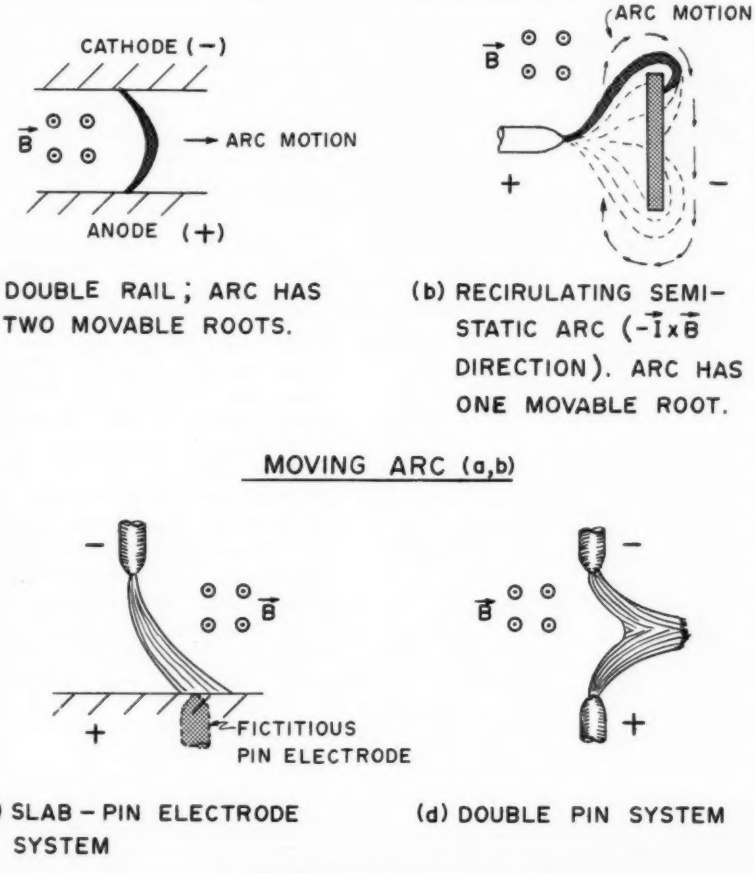
It has been found experimentally that an electric arc subjected to a transverse magnetic field (TMF) normally moves in the $\vec{I} \times \vec{B}$ direction as prescribed by Ampere's law. Given a suitable electrode geometry, the entire arc may continuously advance with both attachment points movable as, for example, along rails (Fig. 1a), or within an annular gap (Refs. 1, 2, 3). It is also possible for only one "end" of the arc to circle about a multisided electrode with the other end anchored; this has only been observed for arc motion in the minus $\vec{I} \times \vec{B}$ direction (Fig. 1b and Ref. 4). The complicated phenomenon of retrograde motion in all its variations will be completely ignored, however, for the remainder of this discussion.

In the (amperian) moving-arc case, the $\vec{I} \times \vec{B}$ force is balanced by a gasdynamic drag force when the arc column has attained a constant average velocity. By way of contrast, an electric arc can also be maintained (or, alternately, can maintain itself) statically in a TMF; this condition can be achieved only with certain electrode-arc attachment configurations. One such arrangement is characterized by an "infinite" slab electrode and a pin electrode (Fig. 1c); the arc column bends under the influence of the magnetic field and retains some final curvature (Ref. 5). The arc roots and arc column are stationary. An arc may also be operated between two pin electrodes (Fig. 1d and Ref. 6); this situation is very similar to the last case. For example, the eventual resting point of the arc attachment on the slab may be considered as the "other" pin electrode; however, the second (and fictitious) pin electrode would not be collinear with its partner. The fact that a curved electric arc can be held stationary in a TMF with either the infinite slab-pin electrode or double-pin electrode system highlights the contrast of these cases with those depicted in Figs. 1a and 1b. In the previous examples, the arc is always free to execute constant or recurrent translatory motion. It is clear that gasdynamic forces oppose the movement of the arc column when it travels in the $+ \vec{I} \times \vec{B}$ direction. Any gasdynamic forces operative on the stationary arcs are not so obvious.

In the static arc cases (Figs. 1c and 1d), the arc is bent but not extinguished by the $\vec{I} \times \vec{B}$ force. This poses the question: what counteracting force or mechanism enables the arc to maintain itself in the TMF? (This question is equally valid for the moving arc, but will not be considered here). Momentum is imparted to the static arc by $\vec{j} \times \vec{B}$ forces, where $\vec{j}$ is the current density. This acquired momentum must somehow be transferred from the arc to its surroundings (e.g., to the cooler gas, to the walls or electrodes), or else the arc would be accelerated and lead to a nonstationary, nonsteady state behavior. One may now distinguish between two extreme cases of momentum transport for the static electric arc in a TMF. Momentum transfer occurs either (a) *across* the



(a) DOUBLE RAIL; ARC HAS TWO MOVABLE ROOTS.

(b) RECIRULATING SEMI-STATIC ARC ($-\vec{I} \times \vec{B}$ DIRECTION). ARC HAS ONE MOVABLE ROOT.

(c) SLAB – PIN ELECTRODE SYSTEM

(d) DOUBLE PIN SYSTEM

Figure 1. Different arc attachment configurations.

surface boundaries of the arc or (b) *along* the arc axis. Two types of flow patterns are associated with these transfer modes–namely, a "jet flow" across the arc column, and an "axial" flow along the arc column. The two momentum exchange modes, together with the flow patterns, are discussed below.

The momentum transfer through the arc surface can only be of a gasdynamic nature since $\vec{j}$ approaches zero at the arc edge. This gasdynamic surface force (momentum transfer per unit time) is given by the force term $\rho\ \vec{v}(\vec{v}\cdot\vec{dA})$ where ρ is the local gas density, $\rho\ \vec{v}$ is the impulse density of the flowing gas, and $\vec{v}\cdot\vec{dA}$ is the gas volume which crosses the surface element per unit time. Since the static arc does not move relative to the outer stagnant gas, frictional and pressure forces do not lead to a resultant transverse force which could balance the $\vec{I} \times \vec{B}$ force. The frictional and pressure forces can therefore be disregarded. Hence, whenever gas crosses the arc boundary, momentum is also transferred from the arc to the surrounding gas, or vice versa. If there is a strong-enough flow leaving the arc, the expelled hot gas generates a bulk reactive force on the arc column. Since the gas may exit across the arc boundary with a rather high velocity (especially at reduced pressure), it is called "jet flow" in deference to the resemblance to the exhaust gases from a jet engine. The reaction force opposes the $\vec{I} \times \vec{B}$ volume force and thus qualitatively answers, at least for the surface boundary mode, the question raised previously. A simplified picture of the jet flow is given in Figure 2a. The detailed explanation of the mechanism of gas acceleration and its reaction effect is given by Schrade (Ref. 7).

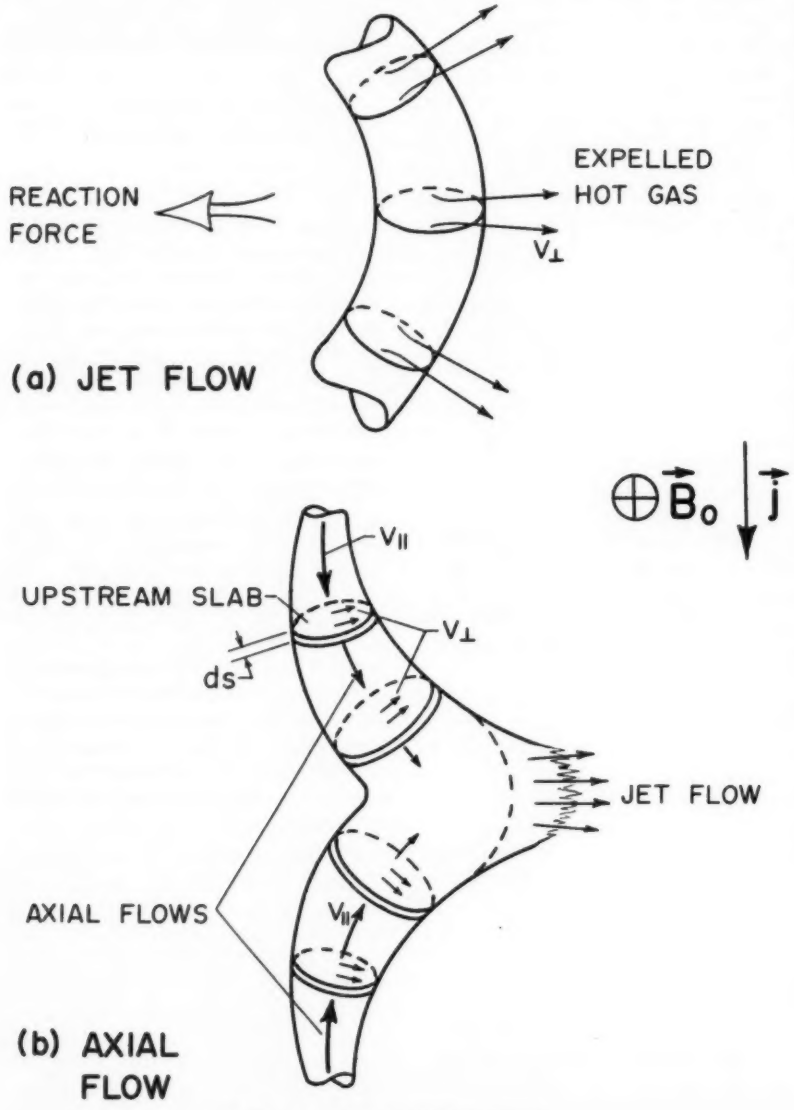


Figure 2. Jet and axial flow patterns.

Momentum transfer *along* the arc axis derives from the fact that radial hydromagnetic forces induce a gas flow along the arc axis (Maecker jet). In order to maintain a static arc, the net transverse momentum gained due to the $\vec{I} \times \vec{B}$ force must be carried away by the axial flow within the arc column. The point is now this: although an axial gas flow can, in general, transfer momentum along the arc axis, it is not immediately apparent how this axial gas flow transports the *transverse* momentum component acquired by the arc. The mechanism involved may be pictured in the following way. The ohmically heated gas of a cross-sectional slab originally located upstream (Fig. 2b), which would cross the arc boundary on the amperian side, is instead carried farther downstream by the axial flow. (As shown in Fig. 2b, two opposing axial flows are operative within the arc column). In contrast to the jet-flow case, gas no longer crosses the arc periphery. The momentum transfer normal to the arc axis is now linked to the fact that the arc diameter increases in the downstream direction. The momentum exchange may be expressed by the relation

$$I B_o = \frac{d}{dt}(M v_\perp) = v_\parallel \frac{d}{ds}(M v_\perp)$$

where M equals the mass of the arc

column per unit length, $v_\perp$ is the average transverse velocity component, and $v_{||}$ is the average velocity component along the arc axis. Assuming $v_\perp$ changes only very slightly with arc length s, one obtains

$$I B_o \simeq v_{||} v_\perp \frac{dM}{ds}$$

This shows that the total mass per unit length, and hence the arc diameter, increases in the direction of the axial flow, since $v_{||}$ decreases downstream and IB_o remains constant. Thus, the transverse momentum goes into increasing the arc diameter in the downstream direction. One should emphasize that, under steady-state conditions, the arc retains its over-all shape. (An argument may be raised here concerning the possibility of a lower temperature in the downstream location; however, this would also lead to an increase of the arc diameter.)

Axial flow can only exist if rather small arc attachment points are present on the electrodes, and if a surface boundary exchange mode is occurring at some *other* point along the arc column. If the arc column is constricted close to the electrodes due to thermal effects, the gas pressure within the column becomes higher than in the center segment of the arc column. The gas is therefore "squeezed" from the two electrode regions toward the midpoint of the arc column; these are the two opposing axial flows indicated before. The two colliding gas streams must eventually break out of the confines of the arc boundaries; thus, one finds that the generally dominant axial flow case is characterized by a small area of jet flow. (A fuller discussion of the heat-conduction losses which lead to the arc constriction close to the electrodes is omitted.) An attempt to illustrate the patterns of jet and axial flow is given in Figure 2; a simple listing of the transfer modes and associated flow patterns is shown in Table I.

TABLE I

Momentum Transfer Mode	Flow Pattern
Across Surface Boundaries	Jet Flow
Along Arc Axis	Axial Flow

It is important to note that many intermediate transfer modes and combinations of flow patterns are possible in a real arc; the surface boundary and axial momentum transfer modes represent, more or less, extreme cases.

The authors wish to thank Mr. F. C. Jarvis of the Aerospace Research Laboratories for the drawings.

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Captain Salvatore F. Giannotta is presently a research physicist with the Thermomechanics Research Laboratory, ARL. He is currently studying magnetic-field interactions with high- and low-density plasmas. He is also responsible for conducting fundamental research in the areas of plasma physics and technology, with special emphasis on plasma-containment techniques in the ultrahigh-temperature regimes.

Captain Giannotta was graduated from Brooklyn College in 1960 with a B.A. in Physics and the academic requirements for a B.S. in Physics. He was commissioned a second lieutenant in the U.S. Air Force Reserve at the same time. Called to active duty in August 1960, he was assigned to New York University for studies in meteorology, after which he served as a weather forecaster in Europe for three years. Subsequently, in 1964, he was assigned to the Air Force Institute of Technology for graduate studies in physics, and was awarded the M.S. in Space Physics in 1966. He continued his graduate studies in Physics at the Ohio State University from 1966 to 1968.

Captain Giannotta is a member of the Air Force Association and the American Physical Society. He has several publications to his credit.

Dr. Herbert O. Schrade has been a Group Leader in the Thermomechanics Research Laboratory, Aerospace Research Laboratories, since 1967. He is currently working on electric-arc discharges in transverse magnetic fields and transverse flows. He was an assistant at the Institute of High Temperature Research, Stuttgart, Germany, from 1958 to 1963. During that time, he received his Diplom. Physicist (M.S. in Physics) in 1958, and his Dr. Science (Ph.D.) in 1963, both from the Technical University, Stuttgart. While working for his Dr. of Science, he studied problems in plasma physics, gas kinetics, and the ohmic heating of gases. Before joining ARL in 1964, he was a research associate at the Ohio State University for one year.

Dr. Schrade is the author and co-author of several publications.

RADIATION AND THE CLASSICAL ELECTRON

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When we accelerate an electron in a force field, the electron emits electromagnetic radiation. How can an electron emit seemingly infinite amounts of radiation—of energy and momentum—without somehow changing? What is the source of this energy? And how can radiation be emitted without some recoil reaction on the electron itself causing it to lose velocity? Somehow, it would seem that conservation laws are being violated. So how do we get around these problems?

Many explanations have been offered since the nature of the electron and radiated power were defined in the late 1890's. The most notable contributions were made by Dirac 30 years ago. Still, no completely satisfactory model of what is happening in the electron-radiation complex of energy and momentum has emerged. Dr. Ronald G. Newburgh of AFCRL considers the problem in his paper, "Radiation and the Classical Electron," in the May 1968 issue of the *American Journal of Physics*, and suggests a new approach. This approach is an extension of Dirac's basic description.

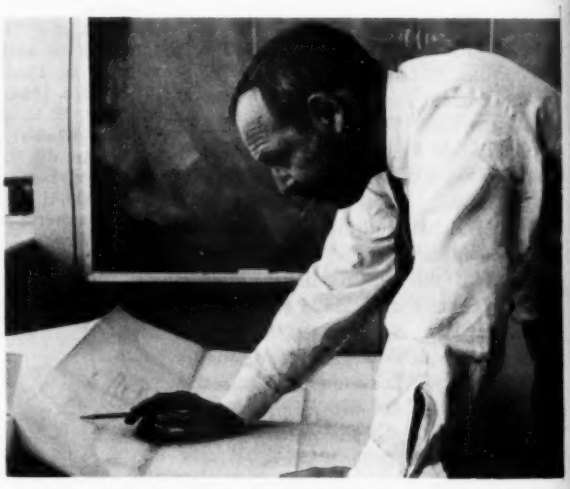
Newburgh devotes most of his paper to a review of relativistic considerations bearing on the problem. In a discussion of inertial frames and Lorentz transformations, he presents the established proofs that the radiated power from the electron is invariant—that is, the power is constant in all inertial frames in which it is measured. He then goes back in time to examine a principle first hinted at by Laue in 1908. Laue showed that the emission of radiation from an oscillating dipole does not affect the dipole velocity. Newburgh has generalized this result and shown that, in spite of the constancy of velocity, there is a momentum change. This change is a result of the mass change of the source due to energy loss. Until the present study, no one had examined the consequences of Laue's insight.

To elaborate on this point, when the electron emits radiation—or, if you prefer, photons—there should be a radiation reaction. There should, in other words, be a transfer of momentum that would cause the electron to lose velocity. However, if velocity is constant, then, to protect conservation laws from abuse, we must assume some loss of mass or energy. From this Laue principle, the radiation reaction can be derived in a very simple fashion.

We start by considering the energy of the particle-radiation system. The expression for the energy of a particle has been treated in a surprisingly cursory fashion in most writings on special relativity. Usually, energy is written as the sum of kinetic plus rest energy. The Laue principle on which Newburgh builds, however, requires a representation of energy which, unlike the sum of kinetic and rest energy, is not stored in the charge, but which is nevertheless radiated away from it. This requires an additional term in the energy equation beyond the kinetic and rest energy. Newburgh suggests that the additional term is found in acceleration energy or Schott energy. (Acceleration energy is energy drawn from the force field itself.) Although this was first proposed by Schott some 50 years ago, it is now seen to be a logical consequence of the Laue principle.

We know that the momentum of any system is expressed as mass times velocity. If the velocity of the accelerated particle does not change, then we can't get around the fact that the expenditure of radiation energy must be credited to particle mass. Newburgh considers the mass equivalent of the acceleration energy as part of the particle system. But, unlike kinetic and rest energy, which are an intrinsic part of the electron, the converted mass of acceleration energy is conceived of as a jacket surrounding the electron. So now we can envision the accelerated particle as being not just the point charge of the electron, but as an extended particle. It is this extended particle that loses mass when radiation is emitted, and not its electron nucleus.

With the loss of mass, the velocity of the total particle complex can remain constant, according to the Laue principle. Nevertheless, this thesis doesn't fully account for the radiation reaction, or recoil, problem as the electron emits photons. Newburgh suggests that the electron does recoil from the emission and thus decelerates. But it decelerates only within the total complex of the extended particle. The center of mass of the total particle itself does not change velocity.



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Prior to joining AFCRL in 1961, Dr. Newburgh was associated with private industry and with the Naval Research Laboratory. Since coming to AFCRL, he has spent one year (1964-65) at the Service D'Aeronomie, Verrieres-Le-Buisson, France, where he worked on the Wheeler-Feynman absorber theory of radiation. He is a member of the American Physical Society, the Optical Society of America, and Sigma Xi, and the author and coauthor of a number of publications.

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